Logarithmic two dimensional spin–1/3 fractional supersymmetric conformal field theories and the two point functions

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Abstract

Logarithmic spin–1/3 superconformal field theories are investigated. The chiral and full two–point functions of two- (or more-) dimensional Jordanian blocks of arbitrary weights, are obtained.

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1 Introduction

According to Gurarie [1], conformal field theories which their correlation functions exhibit logarithmic behaviour may be consistently defined. In some interesting physical theories like polymers [2], WZNW models [3–6], percolation [7], the Haldane-Rezayi quantum Hall state [8], and edge excitation in fractional quantum Hall effect [9], logarithmic correlation functions appear. Also the logarithmic operators can be considered in 2D-magnetohydrodynamic turbulence [10,11,12], 2D-turbulence ([13], [14]) and some critical disordered models [15,16]. Logarithmic conformal field theories for D dimensional case ($D > 2$) has also been studied [17]. In this paper we consider a superconformal extension of Virasoro algebra [18,19] corresponding to three–component supermultiplets, and then, following [20–22], generalize the superconformal field to Jordanian blocks of quasi superconformal fields. We then find the two–point functions of chiral- and full-component fields. It is seen that this correlators are readily obtained through formal derivatives of correlators of superprimary fields, just as was seen in [20–22].

2 Superprimary and quasi-superprimary fields

A chiral superprimary field $\Phi(z,\theta)$ with conformal weight Δ, is an operator satisfying [18]

$$[L_n, \Phi(z,\theta)] = \left[z^{n+1}\partial_z + (n+1)\left(\Delta + \frac{\Lambda}{3}\right) z^n\right]\Phi(z,\theta), \tag{1}$$

$$[G_r, \Phi(z,\theta)] = [z^{r+1/3}\delta_\theta - q^2 z^{r+1/3}\theta^2\partial_z - (3r+1)q^2\Delta z^{r-2/3}\theta^2]\Phi(z,\theta), \tag{2}$$

where θ is a paragrassmann variable, satisfying $\theta^3 = 0$, q is one of the third roots of unity, not equal to one, and δ_θ and Λ, satisfy [18,19]

$$\delta_\theta\theta = q^{-1}\theta\delta_\theta + 1, \tag{3}$$

and

$$[\Lambda, \theta] = \theta, \qquad [\Lambda, \delta_\theta] = -\delta_\theta. \tag{4}$$

Here L_n's and G_r's are the generators of the supervirasoro algebra satisfying

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m}, \\ [L_n, G_r] &= \left(\frac{n}{3} - r\right) G_{n+r}, \end{aligned} \tag{5}$$

and

$$G_rG_sG_t + \text{five other permutations of the indices} = 6L_{r+s+t}. \tag{6}$$

The superprimary field $\Phi(z,\theta)$, is written as

$$\Phi := \Phi(z,\theta) = \varphi(z,\theta) + \theta\varphi_1(z) + \theta^2\varphi_2(z), \tag{7}$$

where $\varphi_1(z)$ and $\varphi_2(z)$ are paragrassmann fields of grades 1 and 2, respectively. One can similarly define a complete superprimary field $\Phi(z,\bar{z},\theta,\bar{\theta})$ with the weights $(\Delta,\bar{\Delta})$ and the expansion

$$\Phi = \sum_{k,k'=0}^{2} \theta^k \bar{\theta}^{k'} \varphi_{kk'}, \tag{8}$$

through (1) and (2), and obvious analogous relations with $\bar{L}_n$'s and $\bar{G}_r$'s. Now suppose that the first component field $\varphi(z)$ in chiral superprimary field $\Phi(z,\theta)$, has a logarithmic counterpart $\varphi'(z)$ [20]:

$$[L_n, \varphi'(z)] = [z^{n+1}\partial_z + (n+1)z^n\Delta]\varphi'(z) + (n+1)z^n\varphi(z). \tag{9}$$

We will show that $\varphi'(z)$ is the first component field of a new superfield $\Phi'(z,\theta)$, which is the formal derivative of the superfield $\Phi(z,\theta)$ with respect to its weight. Let us define the fields $f'_r(z)$ by

$$[G_r, \varphi'(z)] =: z^{r+1/3} f'_r(z), \tag{10}$$

where $r + \frac{1}{3}$ is an integer. Following [22], acting on the both sides of the above equation with L_m and using the Jacobi identity, and using (9), (1), and (5), we have

$$\begin{aligned} [L_m, f'_r(z)] &= \left(\frac{m}{3} - r\right) z^m [f'_{m+r}(z) - f'_r(z)] + \Big[z^{m+1}\partial_z \\ &\quad + (m+1)\left(\Delta + \frac{1}{3}\right) z^m\Big] f'_r(z) + (m+1)z^m\varphi_1(z). \end{aligned} \tag{11}$$

Demanding

$$[L_{-1}, f'_r(z)] = \partial_z f'_r(z), \tag{12}$$

it is easy to shown that

$$f'_r(z) = \begin{cases} \psi'(z), & r \geq -1/3 \\ \psi''(z), & r \leq -4/3. \end{cases} \tag{13}$$

Then, equating $[L_1, f'_{-4/3}(z)]$ and $[L_1, f'_{-7/3}(z)]$, we obtain

$$\psi'(z) = \psi''(z) =: \psi'_1. \tag{14}$$

So in this way we obtain a well–defined field ψ'_1, satisfying

$$[G_r, \varphi'] = z^{r+1/3}\psi'_1, \tag{15}$$

$$[L_n, \psi_1'] = \left[z^{n+1}\partial_z + (n+1)z^n\left(\Delta + \frac{1}{3}\right)\right]\psi_1' + (n+1)z^n\psi_1. \tag{16}$$

Again, let's define the fields $h_r'(z)$ through

$$[G_r, \psi_1']_{q^{-1}} := -z^{r+1/3}h_r'(z). \tag{17}$$

Acting both sides with L_m and using the generalized Jacobi identity [18]:

$$[[G_r, \psi_1']_{q^{-1}}, L_m] + [G_r, [L_m, \psi_1']]_{q^{-1}} + [[L_m, G_r], \psi_1']_{q^{-1}} = 0, \tag{18}$$

we obtain

$$\begin{aligned}[L_m, h_r'] \;&=\; \left[z^{m+1}\partial_z + (m+1)\left(\Delta + \frac{1}{3}\right)z^m\right]h_r' + (m+1)z^m\psi_2 \\ &\quad + \left(\frac{m}{3} - r\right)z^m h_{m+r}' + \left(r + \frac{1}{3}\right)z^m h_r'.\end{aligned} \tag{19}$$

Then, using the same method applied to determine the form of the functions $f_r'(z)$, we find a well–defined field ψ_2' satisfying

$$[G_r, \psi_1']_{q^{-1}} = -z^{r+1/3}\psi_2', \tag{20}$$

$$[L_n, \psi_2'] = \left[z^{n+1}\partial_z + (n+1)\left(\Delta + \frac{2}{3}\right)z^n\right]\psi_2' + (n+1)z^n\psi_2. \tag{21}$$

Finally, we must calculate $[G_r, \psi_2']_{q^{-2}}$. Substituting for $\psi_2'(z)$ from (20) and (15), and using (6), we have

$$\begin{aligned}[G_r, \psi_2']_q \;&=\; -[z^{r+1/3}\partial_z\varphi'(z) + (3r+1)z^{r-2/3}\Delta\varphi'(z) \\ &\quad +(3r+1)z^{r-2/3}\varphi(z)].\end{aligned} \tag{22}$$

Now we define the quasi superprimary field Φ':

$$\Phi' := \Phi'(z, \theta) = \varphi'(z) + \theta\psi_1'(z) + \theta^2\psi_2'(z). \tag{23}$$

It is easy to see that

$$[L_n, \Phi'] = \left[z^{n+1}\partial_z + (n+1)z^n\left(\Delta + \frac{\Lambda}{3}\right)\right]\Phi' + (n+1)z^n\Phi, \tag{24}$$

$$[G_r, \Phi'] = [z^{r+1/3}(\delta_\theta - q^2\theta^2\partial_z) - (3r+1)z^{r-2/3}q^2\Delta\theta^2]\Phi' - q^2(3r+1)z^{r-2/3}\theta^2\Phi. \tag{25}$$

We see that (24) and (25) are formal derivatives of (1) and (2) with respect to Δ, provided one defines the formal derivative [20–22]

$$\Phi'(z, \theta) =: \frac{d\Phi}{d\Delta}. \tag{26}$$

The two superfields Φ and Φ', are a two dimensional Jordanian block of quasi-primary fields. The generalization of the above results to an m dimensional Jordanian block is obvious:

$$[L_n, \Phi^i] = \left[z^{n+1}\partial_z + (n+1)z^n\left(\Delta + \frac{\Lambda}{3}\right)\right]\Phi^i + (n+1)z^n\Phi^{i-1}, \tag{27}$$

and

$$[G_r, \Phi^{(i)}] = [z^{r+1/3}(\delta_\theta - q^2\theta^2\partial_z) - (3r+1)q^2z^{r-2/3}\Delta\theta^2]\Phi^i - q^2\theta^2(3r+1)z^{r-2/3}\Phi^{(i-1)}. \tag{28}$$

Here $1 \leq i \leq m-1$, and the first member of the block, $\Phi^{(0)}$, is a superprimary field. It is easy to show that (27) and (28) are satisfied through the formal relation

$$\Phi^{(i)} = \frac{1}{i!}\frac{d^i\Phi^{(0)}}{d\Delta^i}. \tag{29}$$

3 Two point functions of Jordanian blocks

Consider two Jordanian blocks of chiral quasi-primary fields Φ_1 and Φ_2, with the weights Δ_1 and Δ_2 and dimensions p and q, respectively. As the only closed subalgebra of the super Virasoro algebra the central extension of which is trivial is formed by $[L_{-1}, L_0, G_{-1/3}]$, the correlator of fields with different weights may be nonzero. According to [18],

$$< \varphi_k^(0)\varphi_{k'}'^{(0)} >= a_K\frac{A_{kk'}(\Delta + \Delta' + (k+k'-3)/3)^{B_{kk'}}}{(z-z')^{\Delta+\Delta'+(k+k')/3}} =: a_K f_{k,k'}(z-z'), \tag{30}$$

where $A_{kk'}$ and $B_{kk'}$ are the components of the following matrices:

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ q^2 & -q^2 & q^2 \end{pmatrix}, \tag{31}$$

a_0, a_1, and a_2, are arbitrary constants, and

$$K = k + k' \bmod 3. \tag{32}$$

The general form of the two point functions of Jordanian blocks is then readily obtained, using (29):

$$< \varphi_k^{(i)}\varphi_{k'}'^{(j)} >= \frac{1}{i!}\frac{1}{j!}\frac{d^i}{d\Delta^i}\frac{d^j}{d\Delta'^j}\frac{a_K A_{kk'}(\Delta + \Delta' + (k+k'-3)/3)^{B_{kk'}}}{(z-z')^{\Delta+\Delta'+(k+k')/3}}. \tag{33}$$

Here $0 \leq i \leq p-1$, and $0 \leq j \leq q-1$. In this formal differentiation, one should treat the constants a_i as functions of Δ and Δ'. So, there will be other arbitrary constants

$$a_i^{(j),(k)} := \frac{d^j}{d\Delta^j}\frac{d^k}{d\Delta^k} a_i \tag{34}$$

in these correlators.

To consider the correlators of the full field, one begins with

$$< \varphi_{k\bar{k}}^{(00)}(z,\bar{z})\varphi'^{(00)}_{k'\bar{k}'}(z,\bar{z}) >= a_{K\bar{K}} q^{-k\bar{k}} f_{k,k'}(z-z')\bar{f}_{\bar{k},\bar{k}'}(\bar{z}-\bar{z}'), \tag{35}$$

obtained in [18]. Here $f_{k,k'}(z-z')$ is defind in (30) and $\bar{f}_{\bar{k},\bar{k}'}(\bar{z}-\bar{z}')$ is the same as this with $\Delta \to \bar{\Delta}$ and $\Delta' \to \bar{\Delta}'$. Also,

$$\begin{aligned} K &= k + k' \bmod 3 \\ \bar{K} &= \bar{k} + \bar{k}' \bmod 3. \end{aligned} \tag{36}$$

Using the obvious generalization of (29), it is easy to see that

$$< \varphi_{k,\bar{k}}^{(ij)}\varphi_{k',\bar{k}'}^{(lm)} >= \frac{1}{i!j!l!m!}\frac{d^i}{d\Delta^i}\frac{d^j}{d\bar{\Delta}^j}\frac{d^l}{d\Delta'^l}\frac{d^m}{d\bar{\Delta}'^m}[a_{K\bar{K}} f_{k,k'}\bar{f}\bar{k},\bar{k}']. \tag{37}$$

Again, one should treat $a_{K\bar{K}}$'s as formal functions of the weights, so that differentiating them with respect to the weights introduces new arbitrary parameters.

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